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Deep learning fluid flow reconstruction around arbitrary two-dimensional objects from sparse sensors using conformal mappings

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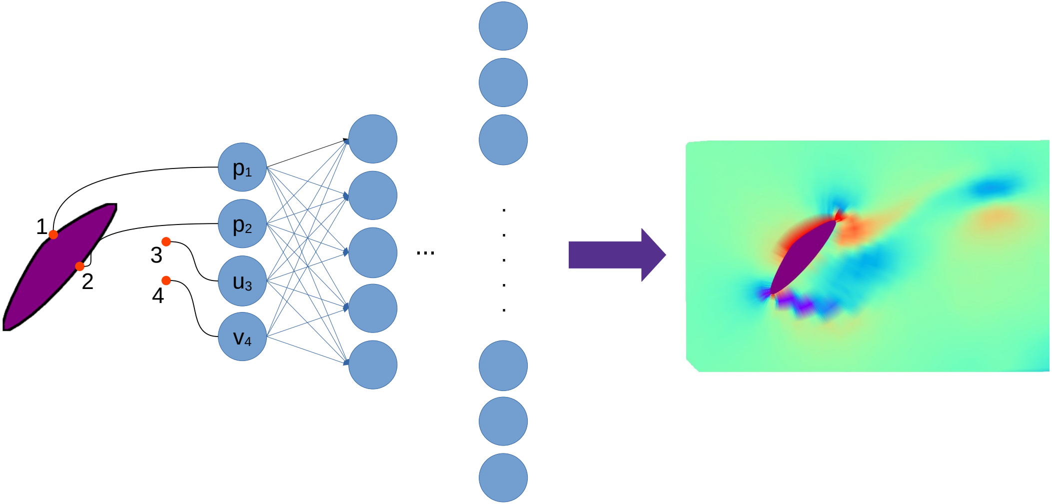
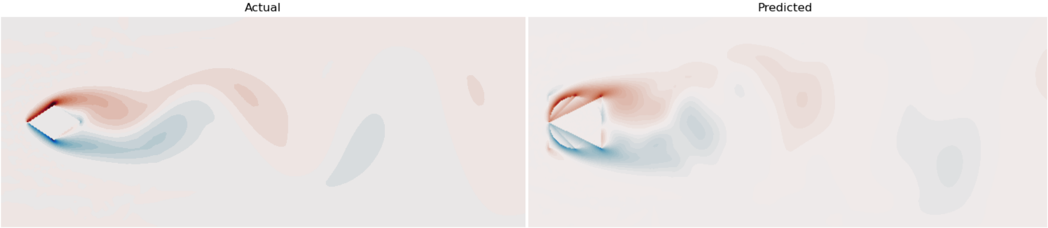
Flow reconstruction from sparse sensors



Introduction and Motivation

- Multi-geometry FR models not as simple as adding data from multiple flow cases to your dataset



- Model shouldn't have to guess the shape of the immersed object

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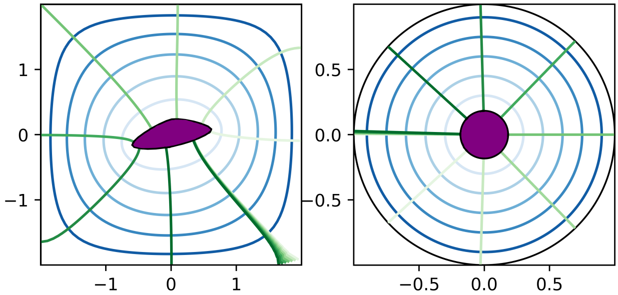
Schwarz-Christoffel Mappings

- Idea: Map all fluid domains to the same shape so no guessing is needed
- Schwarz-Christoffel conformal mappings can be used to do this for 2D



- **Annular sampling** versus **Cartesian sampling**

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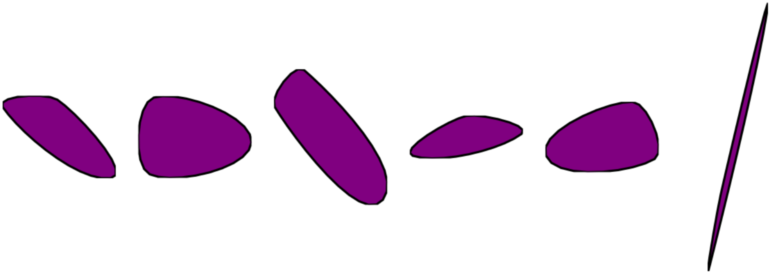
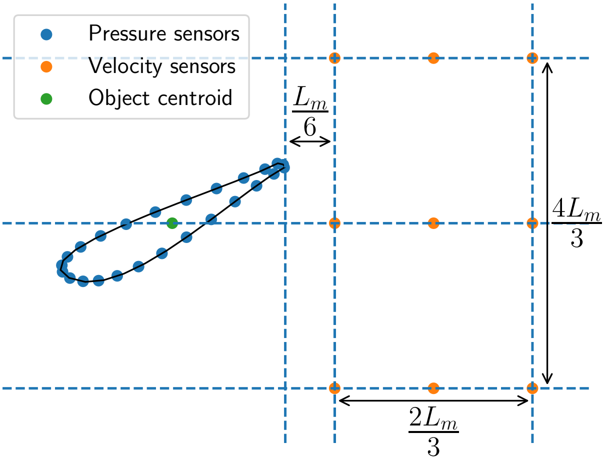
Dataset – Geometries

- 80 random geometries; 64 training and 16 test



- $\mathrm{Re} = 300$ simulations past each geometry using PyFR (implicit LES, artificial compressibility)

Dataset – Sensor setup



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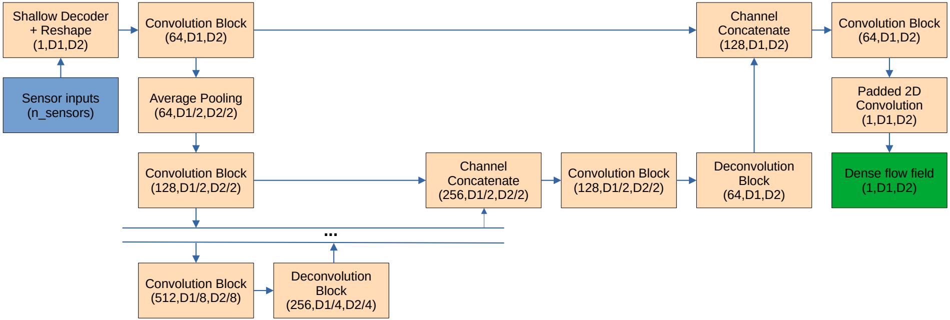
Tasks

- **Spatial multigeometry flow reconstruction (SMGFR)** – sensor measurements and ground truth fields are contemporaneous
- **Spatio-temporal MGFR (STMGFR)** – Ground truth fields are in the future relative to the sensor measurements

Models – Spatial task

- Evaluated various model architectures, one based on the UNet architecture performed best



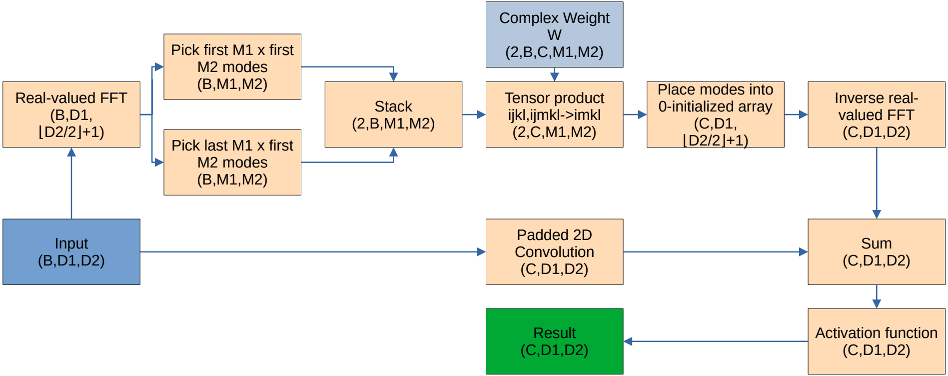
Models – Spatio-temporal task

- Trained using the outputs of the spatial models as inputs. Choose some temporal gap of k timesteps between the input and output snapshots.
- Can be trained for different temporal gaps without re-training the spatial model.
- Set $k = 0$ to use this model as a denoising autoencoder

$$\begin{array}{c}\text{Current sensors}\\ s_{t,i}\end{array} \xrightarrow[\text{(e.g. SD-UNet)}]{\text{Spatial model}} \begin{array}{c}\text{Current field}\\ \hat{x}_{t,i}\end{array} \xrightarrow[\text{(FNO)}]{\text{Temporal model}} \begin{array}{c}\text{Future field}\\ \hat{x}_{t+k,i}\end{array}$$

Models – Spatio-temporal task

- Combine the spatial model with a further model to predict future snapshots from current reconstructed snapshots
- Chosen architecture is a six-layer 64-channel FNO model



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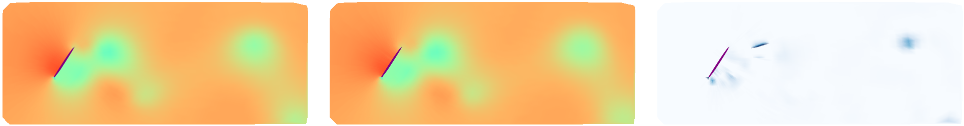
Results – Spatial task – Velocity and Pressure

- Annular sampling enables pressure and velocity components to be estimated with mean absolute percentage errors (MAPE) below 3% and 10%, respectively

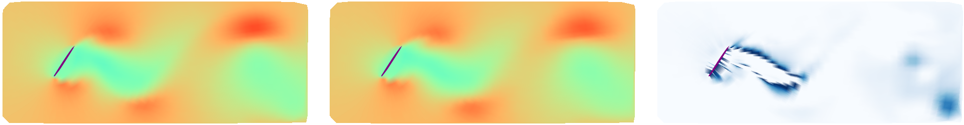
	p		u		v	
	MAE	MAPE	MAE	MAPE	MAE	MAPE
Annular sampling	**0.0118**	**2.43%**	**0.0264**	**8.26%**	**0.0122**	**9.40%**
Cartesian sampling	0.0133	3.32%	0.0332	11.56%	0.0164	15.61%

- Pressure prediction example; target (left), prediction (middle), % error (right):

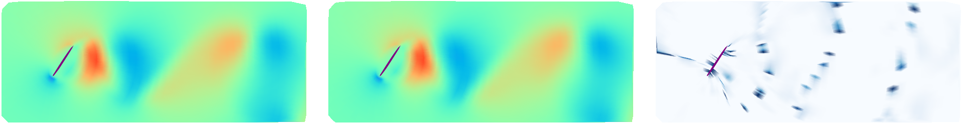


Results – Spatial task – Velocity and Pressure

- *u*-velocity prediction example



- *v*-velocity prediction example



Results – Spatial task – Velocity and Pressure

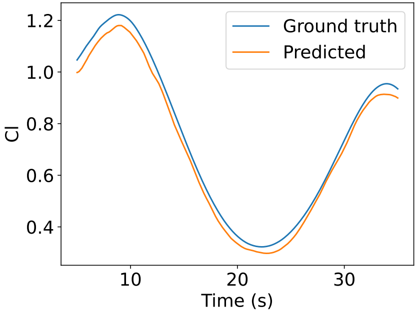
- The pressure and velocity reconstructions can be used to estimate lift and drag via the body force method
- Comparison of the two sampling strategies:

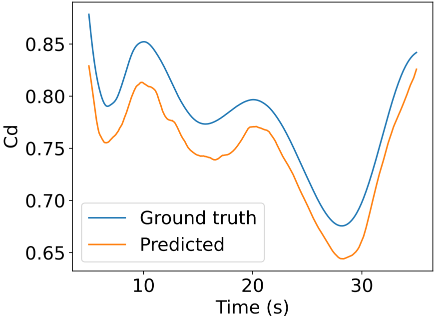
	C_L		C_D	
	MAE	MAPE	MAE	MAPE
Annular sampling	**0.0253**	**4.97%**	**0.0214**	**8.57%**
Cartesian sampling	0.0966	28.18%	0.0684	29.67%

- Annulus sampling removes the need to interpolate variables to object boundaries, greatly boosting accuracy.
- Enables force estimation at $Re = 300$ with MAPE levels comparable to those reported for laminar flow by Chen et al. at $Re = 10$

Results – Spatial task – Velocity and Pressure

- Predicted lift and drag coefficient time evolution for a randomly chosen validation geometry, using Annular sampling





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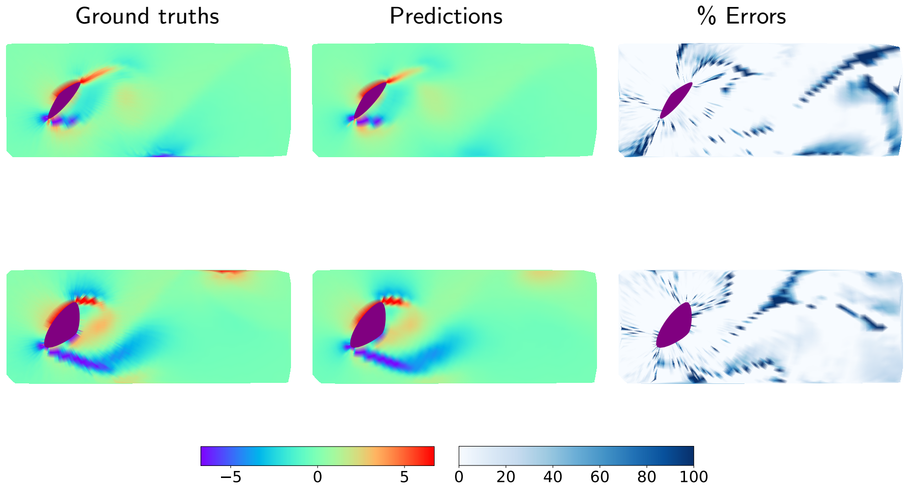
Results – Spatial task – Vorticity

- The reconstruction relationship between velocity/pressure sensor inputs and full vorticity fields is more complicated compared to pressure/velocity full fields
- Hence, percentage error levels are higher:

	SD		SD-UNet	
	MAPE	HVM	MAPE	HVM
Annular sampling	**44.29%**	**34.28%**	**39.92%**	**31.37%**
Cartesian sampling	59.88%	46.14%	47.64%	39.88%

- **High Vorticity MAPE (HVM):** MAPE filtered to not include gridpoints that have ground truth magnitudes under 1% of the peak vorticity magnitude

Results – Spatial task – Vorticity



Results – Spatial task – Vorticity

- The high error levels can be brought down by applying the time-marching FNO model with a $k = 0$ temporal gap, bringing errors down by about $\approx 10\%$:

	SD-UNet+FNO		SD-UNet	
	MAPE	HVM	MAPE	HVM
Annular sampling	28.89%	17.86%	39.92%	31.37%

- In this configuration, the FNO model acts like a denoising autoencoder

Results – Spatio-temporal task – Vorticity

- Input the current full vorticity field into the FNO model and predict the full vorticity field k timesteps in the future
- Future vorticity fields can be predicted with minimal percentage error penalties

k ($\Delta\tau^{*}$ [1])	0 (0.0)		20 (0.667)		80 (2.667)	
	MAPE	HVM	MAPE	HVM	MAPE	HVM
From ground truth	19.76%	10.75%	23.40%	11.75%	29.58%	19.53%
From reconstruction	28.89%	17.86%	31.02%	17.86%	31.88%	21.97%

[1] $\tau * = \tau u_{\infty}/L_m$, where L_m is the length of the side of the box where Bezier curve control points are randomly chosen in

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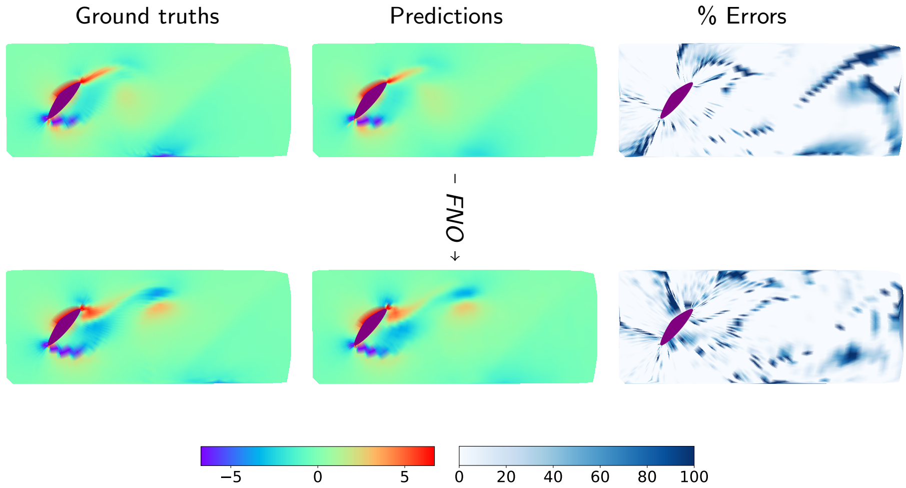
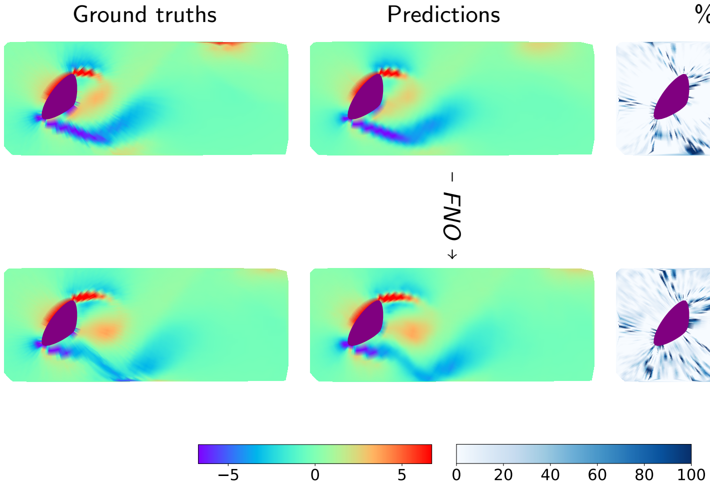
Results – Spatio-temporal task – Vorticity



Results – Spatio-temporal task – Vorticity



Conclusion

- Flow reconstruction around unseen geometries with errors $< 3\%$, $< 10\%$ and $< 30\%$ of p, u and ω
- Predict lift and drag with error levels on the order of 5% and 9% respectively
- Estimate future vorticity fields in the future from current sensor measurements
- Future work:
    - 3D
    - more advanced NN architectures
    - predictions at different Re without retraining

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Cite our work!

- Ali Girayhan Özbay and Sylvain Laizet, "Deep learning fluid flow reconstruction around arbitrary two-dimensional objects from sparse sensors using conformal mappings", AIP Advances 12, 045126 (2022) https://doi.org/10.1063/5.0087488
- First result when you Google "Deep learning flow reconstruction"
- Code repositories:
  - Data generation and training: https://github.com/aligirayhanozbay/flow_prediction
  - Conformal mapping software: https://github.com/aligirayhanozbay/pydscpack